Proceedings of the 363. WE-Heraeus Seminar on: *"Neutron Stars and Pulsars" (Posters and contributed talks)*
Physikzentrum Bad Honnef, Germany, May. 14 – 19, 2006, eds. W. Becker, H.H. Huang, MPE Report 291, pp. 20- 23

Thermal radiation from hot polar cap in pulsars

Janusz Gil[1,2], George Melikidze[1,3], and Bing Zhang[2]

[1] Institute of Astronomy, University of Zielona Góra, Lubuska 2, 65-265, Zielona Góra, Poland
[2] Department of Physics,University of Nevada Las Vegas, Las Vegas, NV, USA
[3] Abasumani Astrophysical Observatory, Al. Kazbegi ave. 2a, 0160, Tbilisi, Georgia

Abstract. Thermal radiation from hot polar caps is examined in radio pulsars with drifting subpulses. It is argued that if these subpulses correspond to sparking discharges of the inner acceleration region right above the polar cap surface then a simple relationship between the observed subpulse drift rate in radio and thermal X-ray luminosity from the polar cap heated by sparks should exist. This relationship is derived and tested in pulsars for which an appropriate good quality data is available.

1. Introduction

Almost 40 years have passed since the discovery of pulsars and the mechanism of their coherent radio emission is still not known. The puzzling phenomenon of drifting subpulses is widely regarded as a powerful tool for the investigation of mechanisms of pulsar radio emission. In the classical model of Ruderman & Sutherland (1975; RS75 henceforth) the spark-associated subbeams of subpulse emission circulate around the magnetic axis due to $\mathbf{E}\times\mathbf{B}$ drift of spark plasma filaments. This model is widely regarded as the most natural and plausible explanation of drifting subpulse phenomenon, at least qualitatively. Despite its popularity the RS75 model is known to suffer from the so-called binding energy problem (for review see Usov & Melrose 1995, 1996). In fact, the cohesive energy of surface iron ions were largely overestimated in RS75 and the "vacuum gap" envisioned by RS75 was impossible to form. A number of attempts have been made to resolve this problem, including a quite exotic proposal that radio pulsars with drifting subpulses are bare polar cap strange stars (BPCSS) rather than neutron stars (Xu, Qiao & Zhang, 1999). Gil & Melikidze, (2002) argued that the formation of RS75 gap above the neutron star polar cap was, in principle, possible, although it required a very strong surface magnetic field, much stronger than the dipolar component inferred from the observed spin-down rate. Growing evidence of such strong non-dipolar surface field accumulates in the literature, both observational and theoretical (see Urpin & Gil 2004, for short review).

Even if the RS75 gap was possible to form, it would not automatically solve the mystery of drifting subpulses. It is well known that the original model of RS75 predicts too fast a drifting rate (e.g. Deshpande & Rankin, 1999; DR99 henceforth). Motivated by this issue Gil, Melikidze & Geppert (2003; GMG03 henceforth) developed further the idea of the inner acceleration region above the polar cap by including the partial screening due to thermionic ions flow from the surface heated by sparks. We will call this kind of the inner acceleration region the "partially screened gap" (PSG henceforth). The slow drift corresponds to $\mathbf{E}\times\mathbf{B}$ drift of spark generated electron-positron pairs, until the total charge density within the partially screened gap reaches the co-rotational value. Since the PSG potential drop is much lower than in the RS75 model, the intrinsic drift rate is compatible with observations (for details see GMG03).

A distinguishing property of PSG model is relatively high predicted heating rate of the polar cap surface, compatible with observations (in contrast to RS75 gap, which overheated the polar cap). The space charge limited model (Arons & Scharleman, 1979, AS79 henceforth; Zhang & Harding, 2000; Harding & Muslimov, 2002, HM02 henceforth) predicts a much lower polar cap heating rate. On the other hand, in the BPCSS model (Xu, Qiao, & Zhang, 1999) no hot spot is expected due to the high thermal conductivity at the BSS surface. Thus, measuring the thermal X-ray luminosity from heated polar caps can potentially reveal the nature of the inner acceleration region in pulsars. This can help us to understand a mechanism of drifting subpulses, which appears to be a common phenomenon in radio pulsars. Recently Weltevrede, Edwards, & Stappers (2006, WES06 henceforth) presented results of a systematic, unbiased search and found that the fraction of pulsars showing drifting subpulses is at least 55 %. They concluded that the conditions for drifting mechanism to work cannot be very different from the emission mechanism of radio pulsars.

In order to test different available models of inner acceleration region in pulsars Zhang, Sanwal & Pavlov 2005; ZSP05 henceforth) observed the best studied drifting subpulse radio pulsar PSR B0943+10 with the *XMM-Newton* observatory. Their observations were consistent with PSG formed in strong, non-dipolar magnetic field just above the surface of very small polar cap. Recently, Kargaltsev, Pavlov & Garmire (2006, KPG06 henceforth) observed the X-ray emission from the nearby PSR B1133+16 and found that this case is also consistent with the thermal radiation from a small hot spot (again much smaller than the canonical polar cap). PSR B1133+16 is almost a twin of PSR B0943+10 in terms of P and $\dot{P}$ values and, interestingly, both these pulsars have very similar X-ray signatures, in agreement with our PSG model (see Table 1).

In this paper we generalized the treatment of ZSP05 and developed detailed model for thermal X-ray emission from radio drifting pulsars. The model matches the observations of PSRs B0943+10 and B1133+16 well, both in radio and X-rays. There is a number of pulsars with measured thermal X-ray radiation from small hot polar cap but unfortunately their drifting properties are not yet known. This is likely to change in the near future due to new sophisticated methods for analysis of intensity fluctuations in weak pulsars being developed (e.g. WRS06 and references therein).

2. PSG model of the inner acceleration region

As already mentioned, growing evidence appears, both observationally and theoretically, that the actual surface magnetic field B_s is highly non-dipolar. Its magnitude can be described in the form $B_s = bB_d$ (Gil & Sendyk 2000; GS00 henceforth), where the enhancement coefficient $b > 1$ and $B_d = 2 \times 10^{12}(P\dot{P}_{-15})^{1/2}$G is the canonical, star centered dipolar magnetic field, P is the pulsar period and $\dot{P}_{-15} = \dot{P}/10^{-15}$ is the period derivative.

The polar cap is defined as the locus of magnetic field lines that penetrate the so-called light cylinder. Conventionally, the polar cap radius $r_{pc} = 1.45 \times 10^4 P^{-0.5}$ cm, and its surface area $A_{pc} \sim 2 \times 10^8 P^{-1}$ cm^2. In the case of non-dipolar surface field the polar cap area must shrink due to flux conservation of the open field lines $A_{pc}B_d = A_pB_s$. Thus, the surface area of the actual polar cap $A_p = b^{-1}A_{pc}$, regardless of the actual shape of the polar cap. Consequently, one can write the polar cap radius in the form $r_p = b^{-0.5}r_{pc}$ (GS00), realizing however that this is only a characteristic dimension of the actual polar cap. First, the canonical expression for the radius of the polar cap (GJ69) is only a geometrical approximation (e.g. Michel 1973). Secondly, the presence of strong surface magnetic field anomalies should make the shape of the polar cap quite irregular.

The charge depleted inner acceleration region above the polar cap results from the deviation of a local charge density ρ from the co-rotational charge density (Goldreich & Julian 1969) $\rho_{\rm GJ} = -\mathbf{\Omega} \cdot \mathbf{B}_s/2\pi c \approx B_s/cP$. For isolated neutron stars one might expect the surface to consist mainly of iron formed at the neutron star's birth (e.g. Lai 2001). Therefore, the charge depletion above the polar cap can result from bounding of the positive $^{56}_{26}$Fe ions (at least partially) in the neutron star surface. As demonstrated by recent exact calculations of Medin & Lai (2006), the iron chains are strongly bound in magnetic field close to 10^{14} G. This can only occur if the surface magnetic field in actual pulsars is dominated by strong non-dipolar components. If this is really possible, then due to significant bounding the positive charges cannot be supplied at the rate that would compensate the inertial outflow through the light cylinder. As a result, a significant part of the unipolar potential drop develops above the polar cap, which can accelerate charged particles to relativistic energies and power the pulsar radiation mechanism.

The ignition of cascading production of electron-positron plasma is crucial for limitation of growing gap potential drop. The accelerated positrons will leave the acceleration region, while the electrons will bombard the polar cap surface, causing a thermal ejection of ions. This thermal ejection will cause partial screening of the acceleration potential drop ΔV corresponding to a shielding factor $\eta = 1 - \rho_i/\rho_{\rm GJ}$ (see GMG03 for details), where ρ_i is charge density of ejected ions, $\Delta V = \eta(2\pi/cP)B_sh^2$ is the potential drop and h is the height of the acceleration region. The gap potential drop is completely screened when the total charge density $\rho = \rho_i + \rho_+$ reaches the corotational value ρ_{GJ}.

GMG03 argued that the actual potential drop ΔV should be thermostatically regulated and the quasi-equilibrium state should be established, in which heating due to electron bombardment is balanced by cooling due to thermal radiation. The quasi-equilibrium condition is $Q_{cool} = Q_{heat}$, where $Q_{cool} = \sigma T_s^4$ is a cooling power surface density by thermal radiation from the polar cap surface and $Q_{heat} = \gamma m_e c^3 n$ is heating power surface density due to back-flow bombardment, $\gamma = e\Delta V/m_ec^2$ is the Lorentz factor, $n = n_{GJ} - n_i = \eta n_{GJ}$ is the number density of back-flowing plasma particles depositing their kinetic energy at the polar cap surface, η is the shielding factor, n_i is the charge number density of thermionic ions and $n_{GJ} = \rho_{GJ}/e = 1.4 \times 10^{11} b\dot{P}_{-15}^{0.5}P^{-0.5}\mathrm{cm}^{-3}$ is the corotational charge number density. It is straightforward to obtain an expression for the quasi-equilibrium surface temperature in the form

$$T_s = (6.2 \times 10^4 \mathrm{K})(\dot{P}_{-15}/P)^{1/4}\eta^{1/2}b^{1/2}h^{1/2}. \quad (1)$$

Let us now interrelate the accelerating potential drop ΔV and the perpendicular (with respect of the magnetic field lines) electric field ΔE which causes $\mathbf{E} \times \mathbf{B}$ drift. Following the original method of RS75 we can argue that the tangent electric field is strong only at the polar cap boundary where $\Delta E = 0.5\Delta V/h = \eta(\pi/cP)B_sh$ (see Appendix A in GMG03 for details). Due to the $\mathbf{E} \times \mathbf{B}$ drift the discharge plasma performs a slow circumferential motion with velocity $v_d = c\Delta E/B_s = \eta\pi h/P$. The time interval to make one full revolution around the polar cap boundary is $\hat{P}_3 \approx 2\pi r_p/v_d$. One then has

$$\frac{\hat{P}_3}{P} = \frac{r_p}{2\eta h}. \qquad (2)$$

If the plasma above the polar cap is fragmented into filaments (sparks) which determine the intensity structure of the instantaneous pulsar radio beam, then in principle, the tertiary periodicity $\hat{P}_3$ can be measured/estimated from the pattern of the observed drifting subpulses (Deshpande & Rankin 1999, Gil & Sendyk 2003). According to RS75, $\hat{P}_3 = NP_3$, where N is the number of sparks contributing to the drifting subpulse phenomenon observed in a given pulsar and P_3 is the primary drift periodicity (distance between the observed subpulse drift bands). On the other hand $N \approx 2\pi r_p/2h$ (GS00). Thus, one can write the shielding factor in the form $\eta \approx (1/2\pi)(P/P_3)$, which depends only on an easy-to-measure primary drift periodicity. Apparently, the shielding parameter η should be much smaller than unity.

The X-ray thermal luminosity is $L_x = \sigma T_s^4 \pi r_p^2 = 1.2 \times 10^{32}(\dot{P}_{-15}/P^3)(\eta h/r_p)^2$ erg/s, which can be compared with the spin-down power $\dot{E} = I\Omega\dot{\Omega} = 3.95 I_{45} \times 10^{31}\dot{P}_{-15}/P^3$ erg/s, where $I = I_{45}10^{45}$g cm^2 is the neutron star moment of inertia (bellow we assume that $I_{45} = 1$). Using equation (2) we can derive the thermal X-ray luminosity and its efficiency as $L_x = 2.5 \times 10^{31}(\dot{P}_{-15}/P^3)(P/\hat{P}_3)^2$, or in the simpler form representing the efficiency with respect to the spin-down power

$$\frac{L_x}{\dot{E}} = 0.63\left(\frac{P}{\hat{P}_3}\right)^2, \qquad (3)$$

which is very useful for comparison with observations. One should realize that this equation holds only for thermal X-rays from hot spot and cannot be applied neither to cooler radiation from the entire stellar surface nor to the magnetospheric component.

We can see that L_x in these equations depends only on radio observables. It is particularly interesting and important that both equations above do not depend on details of the sparking gap model (η, b, h). Although one has to be careful whether all our assumptions are satisfied in real pulsars, it seems that we have found a very useful, relatively easy testable (at least for the order-of-magnitude) relationship between the properties of drifting subpulses observed in radio band and the characteristics of thermal X-ray emission from the polar cap heated by sparks associated with these subpulses. For PSRs B0943+10 and B1133+16, which are the only two pulsars in which both $\hat{P}_3$ and L_x are measured/estimated (Table 1), the above equation holds very well. Interestingly, in both cases $L_x/\dot{E} \sim 10^{-3}$.

Using equation (2) we can write the polar cap temperature in the form

$$T_s = (5.1 \times 10^6\mathrm{K}) b^{1/4}\dot{P}_{-15}^{1/4}P^{-1/2}\left(\frac{\hat{P}_3}{P}\right)^{-1/2}, \qquad (4)$$

where the enhancement coefficient $b = B_s/B_d \approx A_{pc}/A_{bol}$, $A_{pc} = \pi r_{pc}^2$ and $A_{bol} = A_p$ is the actual emitting surface area (bolometric). Since A_{bol} can be determined from the black-body fit to the spectrum of the observed hot-spot thermal X-ray emission, the above equations can be regarded as independent of details of the sparking gap model and depending only on combined radio and X-ray data, similarly as in equations (3).

3. Comparison with observational data

Table 1 presents the observational data and predicted values of a number of quantities for two pulsars, which we believe show clear evidence of thermal X-ray emission from the spark heated polar caps as well as they have known values of tertiary subpulse drift periodicity. The predicted value of $\hat{P}_3$ and/or L_x were computed from eq.(4), while the predicted values of T_s were computed from eq.(5), with $b = A_{pc}/A_{bol}$ determined observationally. The actual surface magnetic field $B_s = bB_d$, where B_d is the canonical dipole magnetic field at the polar cap. As a principle, the predicted (Pred) values were obtained from observational (Obs) values through Eq.(3).

PSR B0943+10. This is the best studied drifting subpulse radio pulsars with $P = 1.09$ s, $\dot{P}_{-15} = 3.52$, $\dot{E} = 10^{32}$ erg s^{-1}, $P_3 = 1.86P$, $\hat{P}_3 = 37.4P$ and $N = \hat{P}_3/P_3 = 20$ (DR99). It was observed by ZSP05, who obtained an acceptable thermal BB fit with bolometric luminosity $L_x = (5^{+0.6}_{-1.6}) \times 10^{28}$ erg s^{-1} and thus $L_x/\dot{E} = (0.49^{+0.06}_{-0.16}) \times 10^{-3}$. The bolometric polar cap surface area $A_{bol} = 10^7[T_s/(3 \times 10^6\mathrm{K})]^{-4}\mathrm{cm}^2 \sim (1^{+4.0}_{-0.4}) \times 10^7$ cm^2 is much smaller than the conventional polar cap area $A_{pc} = 6 \times 10^8$ cm^2. This all correspond to the best fit temperature $T_s \sim 3.1 \times 10^6$ K (see Fig. 1 in ZSP05). The predicted value of $L_x/\dot{E}$ calculated from equation (3) agrees very well with the observational data. The surface temperature T_s calculated from equation (4) with $b = A_{pc}/A_{bol}$ is also in good agreement with the best fit. The shielding factor $\eta = 0.09$, thus more than 90 % of the available vacuum potential drop is screened.

PSR B1133+16. This pulsar with $P = 1.19$ s, $\dot{P}_{-15} = 3.7$, and $\dot{E} = 9 \times 10^{31}$ erg s^{-1} is almost a twin of PSR B0943+10. KPG06 observed this pulsar with Chandra and found an acceptable BB fit $L_x/\dot{E} = (0.77^{+0.13}_{-0.15}) \times 10^{-3}$, $A_{bol} = (0.5^{+0.5}_{-0.3}) \times 10^7$ cm^2 and $T_s \approx 2.8 \times 10^6$ K. These values are also very close to those of PSR B0943+10, as should be expected for twins. Using equation (3) we can predict $\hat{P}_3/P = 27^{+5}_{-2}$ for B1133+16. Interestingly, Nowakowski (1996) obtained fluctuation spectrum for this pulsar with clearly detected long period feature corresponding to about $32P$. Most recently, WES06 found $P_3/P = 3 \pm 2$ and long period feature corresponding to $(33 \pm 3)P$ in the fluctuation spectrum of PSR B1133+16. The latter value seem to coincide with that of Nowakowski (1996), as well as with our predicted range of $\hat{P}_3$. We therefore claim that this is the actual tertiary periodicity in

Table 1. Comparison of observed and predicted parameters of thermal emission from hot polar caps

Name	P_3/P	$\hat{P}_3/P$		$L_x/\dot{E}\times 10^3$		b	$T_s^{(\mathrm{obs})}$	$T_s^{(\mathrm{pred})}$	B_d	B_s
PSR B	Obs.	Obs.	Pred.	Obs.	Pred.	$A_\mathrm{pc}/A_\mathrm{bol}$	10^6 K	10^6 K	10^{12}G	10^{14}G
$0943+10$	1.86	37.4	36^{+8}_{-2}	$0.49^{+0.06}_{-0.16}$	0.45	60^{+140}_{-48}	$3.1^{+0.9}_{-1.1}$	$3.3^{+1.2}_{-1.1}$	3.95	$2.37^{+5.53}_{-1.90}$
$1133+16$	3^{+2}_{-2}	(33^{+3}_{-3})	27^{+5}_{-2}	$0.77^{+0.13}_{-0.15}$	$0.58^{+0.12}_{-0.09}$	$11.1^{+16.6}_{-5.6}$	$2.8^{+1.2}_{-1.2}$	$2.1^{+0.5}_{-0.4}$	4.25	$0.47^{+0.71}_{-0.24}$

PSR B1133+16 and show it in paranthesis in Table 1. It is worth noting that $\hat{P}_3/P_3 = 33 \pm 3$ is quite close to 37.4 measured in the radio twin PSR B0943+10. Note also that the number of sparks predicted from our hypothesis is $N = \hat{P}_3/P_3 = (33 \pm 3)/(3 \pm 2) = 11^{+25}_{-6}$, so it can also be close to 20, as in the case of twin PSR B0943+10. The shielding factor $\eta = 0.05^{+0.11}_{-0.02}$. Thus, only several % of the vacuum gap potential drop is available for acceleration.

4. Conclusions and discussion

Within the partially screened gap model of the inner acceleration region in pulsars developed by GMG03 we derived a simple relationship between the X-ray luminosity L_x from the polar cap heated by sparks and the tertiary periodicity $\hat{P}_3$ of the spark-associated subpulse drift observed in radio band. In PSRs B0943+10 and B1133+16 for which both L_x and $\hat{P}_3$ are known, the predicted relationship between observational quantities holds quite well.

Both the heating and the drifting rate depends on the gap potential drop. In this paper we made the point that there is continuum of cases between pure vacuum gap and the space charge limited flow. The original RS75 model predicts much too high a subpulse drift rate and an X-ray luminosity. Other available acceleration models predict too low a luminosity and the explanation of drifting subpulse phenomenon is generally not clear (see ZSP05 for more detailed discussion). Approximately, the bolometric X-ray luminosity for the space charge limited flow (Arons & Sharleman 1979) is about $(10^{-4} \div 10^{-5})\dot{E}$ (Harding & Muslimov 2002), and for the pure vacuum gap (RS75) is about $(10^{-1} \div 10^{-2})\dot{E}$ (ZSP05), while for the partially screened gap (GMG03) is $\sim 10^{-3}\dot{E}$ (this paper). The latter model also predicts right $\mathbf{E}\times\mathbf{B}$ plasma drift rate. Thus, combined radio and X-ray data are consistent only with the partially screened gap model, which requires very strong (generally non-dipolar) surface magnetic fields. Observations of the hot-spot thermal radiation almost always indicate bolometric polar cap radius much smaller than the canonical value (by an order of magnitude). Most probably such a significant reduction of the polar cap size is caused by the flux conservation of the non-dipolar surface magnetic fields connecting with the open dipolar magnetic field lines at distances much larger than the neutron star radius. A small part of this reduction can follow from the fact that discrete sparks do not heat the entire polar cap area (say about a half), but uncertainties in determination the size of the polar cap are likely to account for this effect.

Acknowledgements. We gratefully acknowledge the support by the WE-Heraeus foundation. We also acknowledge the support of the Polish State Committee for scientific research under Grant 1 P03D 029 26.

References

Arons, J., Sharleman,E.T. 1979, ApJ, 231, 854 235,576(CR80)
Deshpande, A.A., & Rankin,J.M. 1999, ApJ, 524, 1008 (DR99)
Gil, J., & Sendyk, M., 2000, ApJ, 541, 351 (GS00)
Gil J. & Melikidze G.I., 2002, ApJ, 577,909
Gil, J., & Sendyk, M. 2003, ApJ, 585
Gil J., Melikidze G.I., & Geppert U., 2003, A&A, 407, 315 (GMG03)
Goldreich P., & Julian H., 1969, ApJ, 157,869
Harding, A, & Muslimov,A. 2002, ApJ, 568, 862
Kargaltsev,O., Pavlov, G.G., & Garmire, G.P. 2006, ApJ, 636, 406(KGP06)
Lai D., 2001, Rev. Mod. Phys., 73, 629
Nowakowski,L., 1996, ApJ 457, 868
Medin, Z., & Lai D., 2006, astro-ph/0607277
Michel, F.C., 1973, ApJ 180,207
Ruderman M.A., Sutherland P.G., 1975, ApJ, 196, 51 (RS75)
Urpin V., & Gil J., 2004, A& A, 415, 356
Usov, V.V., & Melrose, D.B.,1995, Australian J. Phys., 48, 571
Usov, V.V., & Melrose,D.B., 1996, ApJ 464, 306
Weltevrede P., Edwards R.I., Stappers B.A., 2005, A&A, 445, 243 (WES06)
Xu, R., Qiao, G.J., & Zhang, B. 1999, ApJ, 522, L109
Zhang B., Sanwal D., & Pavlov G.G., 2005, ApJ, 624, L109 (ZSP05)
Zhang B., Harding A., & Muslimov A., 2000, ApJ, 531, L135
Zhang, B., & Harding, A. 2000, ApJ, 532, 1150